The 19th International Conference on Finite or Infinite Dimensional Complex Analysis and Applications

Aster Plaza
Hiroshima, Japan

December 11 – 15, 2011

Program and Abstracts

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Schedule Overview

Sunday, Decembner 11th

14:00-	**Registration**	
	Room I & II	
15:00-15:15	**Opening Address**	
15:15-16:15	**Stankewitz**	
16:30-17:30	**Yamanoi**	
18:00-20:00	**Opening Reception** at the restaurant Rencontre	

Monday, December 12th

	Room I & II	
09:15-10:15	**Kamimoto**	
(**Tea**)		
10:45-11:45	**Taniguchi**	
(**Lunch**)		
	Room I	Room II
13:20-13:50	**Maitani**	**KH Shon**
13:55-14:25	**Jaerisch**	**Fujimura**
14:30-15:00	**Umemoto**	**Shimauchi**
(**Tea**)		
15:30-16:00	**Komori**	**JS Choi**
16:05-16:35	**Fujikawa**	**Liao**
16:40-17:10	**Matsuzaki**	**Sumi**
17:15-17:45	**Nakamura**	**Okuyama**

Tuesday, December 13th

	Room I & II	
09:00-10:00	**Porter**	
10:10-11:10	**Koo**	
11:10-	**Excursion** to Miyajima Island	

Wednesday, December 14th

	Room I & II	
09:15-10:15	**Korhonen**	
(**Tea**)		
10:45-11:45	**Yoshino**	
(**Lunch**)		
	Room I	Room II
13:20-13:50	**Coulembier**	**Hirata**
13:55-14:25	**Nishihara**	**Ohno**
14:30-15:00	**Le Hung Son**	**Sugawa**
(**Tea**)		
15:30-16:00	**Hitzer**	**YC Kim**
16:05-16:35	**Sekiguchi**	**Makhmutov**
16:40-17:10	**Honda**	**Denega**
17:15-17:45	**Kou-Ou**	**Vasiliev**

Thursday, December 15th

	Room I & II	
09:15-10:15	**Stoll**	
(**Tea**)		
10:45-11:45	**KT Kim**	
(**Lunch**)		
	Room I	Room II
13:20-13:50	**JC Joo**	**Mizuta**
13:55-14:25	**HJ Lee**	**Itoh**
14:30-15:00	**Kikuta**	**YJ Lee**
(**Tea**)		
15:30-16:00	**AR Seo**	**Tanaka**
16:05-16:35	**Aihara**	**KG Na**
16:40-17:10	**HS Kim**	**Rathie**
17:15-17:45	**Kato**	**Zelinskyi**
18:30-20:30	**Banquet** at the restaurant Kurikawa	

Plenary Lectures

Newton polyhedra and oscillatory integrals

Joe Kamimoto

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In this talk, the asymptotic behavior at infinity of oscillatory integrals is in detail investigated by using the Newton polyhedra of the phase and the amplitude. We are especially interested in the case that the amplitude has a zero at a critical point of the phase. The properties of poles of local zeta functions, which are closely related to the behavior of oscillatory integrals, are also studied under the associated situation.

Joint work with Koji Cho and Toshihiro Nose (Kyushu University).

References

[1] K. Cho, J. Kamimoto and T. Nose: Asymptotic analysis of oscillatory integrals via the Newton polyhedra of the phase and the amplitude, To appear in J. Math. Soc. Japan.

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On the generalization of Forelli's theorem in several complex variables

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The statement of classic theorem of F. Forelli proved in 1977 is as follows:

Theorem 1 *If a complex-valued function f from the open unit ball in n dimensional complex Euclidean space satisfies the two conditions:*
(1) *f admits a formal Taylor expansion at the origin* 0, *and*
(2) *f is holomorphic along every straight complex line passing through* 0,
then f is holomorphic on the unit ball.

As one sees from the statement, this apparently is a significant variation of the celebrated Hartogs' analyticity theorem in several complex variables. On the other hand, this theorem is well-known to be highly subtle, in the sense that the conditions in the hypothesis of the theorem was regarded impossible to be improved by many experts. One aspect is clear; if the birational blow-up method is applied at the origin, the new setting deals with the function that is holomorphic along one dimensional complex lines spanned along the exceptional fiber and does not immediately show any further holomorphicity in other directions. Thus the existence of the formal Taylor series at the origin must be truly the special condition. Thus it was not surprising that there have been no significant report on generalization of Forelli's theorem for almost 30 years, perhaps due to such difficulty. After the long "drought", there have been two, as far as we are aware of, recent generalizations, which are distinguished from each other. Both concern the generalization of the condition (2). The first was the two-dimensional theorem by E. Chirka (2006) replacing the condition (2) by the concept of smooth "singular" foliation at the origin by holomorphic curves. The other one is all-dimensional, but the condition (2) was replaced by the condition of annihilation of f by the conjugate of a contracting holomorphic vector field without resonance, which is by K.T. Kim, E. Poletsky and G. Schmalz (2008). These two conditions represent two mutually independent cases. Then, a recent (2011) result by J.C. Joo, K.T. Kim and G. Schmalz shows that, with a slightly stronger condition, the all dimensional version of Chirka's theorem is true, thus answering the question raised by Chirka in his paper "Variations of Hartogs' theorem". I shall start with, in the lecture, the history and survey of this circle of research first, then present sketch of proofs of the major results, and finally finish with some open problems for possible future research.

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Difference versions of the Painlevé equations

Risto Korhonen

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An ordinary differential equation is said to possess the Painlevé property if all of its solutions are single-valued about all movable singularities. Painlevé and his colleagues analyzed a large class of second-order differential equations rejecting those equations which did not have the Painlevé property. They singled out a list of 50 equations out of which there were six which could not be integrated in terms of known functions. These equations are now known as the Painlevé differential equations.

Ablowitz, Halburd and Herbst have suggested that the existence of sufficiently many finite-order meromorphic solutions is a good candidate for discrete Painlevé property. In [2] we showed that if the difference equation

$$w(z+1) + w(z-1) = R(z, w(z)), \tag{1}$$

where $R(z, w(z))$ is rational in $w(z)$ and meromorphic in z, has just one or more finite-order meromorphic solutions that grow faster than the coefficients of the equation, then either $w(z)$ satisfies a difference linear or Riccati equation or else equation (1) can be transformed to one of a list of canonical difference equations. This list consists of all known difference Painlevé equations of the form (1), together with their autonomous versions. This suggests that the existence of finite-order meromorphic solutions is a good detector of integrable difference equations. The finite-order growth condition was replaced by a weaker condition in [1], where we also showed that this condition is essentially the best possible, at least in the first-order case.

Joint work with Rod Halburd (University College London) and Kazuya Tohge (Kanazawa University).

References

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Difference of composition operators

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In this talk we discuss the boundedness and the compactness of difference of composition operators. We first discuss known results on the unit disc and the polydisc, then we focus on the Fock-Sobolev spaces. Linear combinations of composition operators acting on the Fock-Sobolev spaces of several variables will be discussed. We show that such an operator is bounded only when all the composition operators in the combination are bounded individually. So, cancelation phenomenon is not possible on the Fock-Sobolev spaces, in contrast to what have been known on other well-known function spaces over the unit disk. We also show the analogues for compactness and the membership in the Schatten classes. In particular, compactness and the membership in some/all of the Schatten classes turn out to be the same.

Joint work with Hong Rae Cho(Pusan University) and Boo Rim Choe(Korea University)

References

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Numerical solution of the Beltrami equation

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We will survey some of the existing methods for solving the Beltrami equation $f_{\overline{z}} = \mu f_z$ numerically. The solution is a "μ-conformal" mapping of prescribed domains, such as the unit disk. The ideas behind these methods (for example, singular integrals or circle packings) will be discussed.
A simpler and more intuitive method for solving the Beltrami equation will be presented, which may be described briefly (and incompletely) as follows: triangulate D_1, map each triangle in a piecewise affine way to form a domain D_2' which is μ-conformal to D_1, and then map D_2' conformally to the desired image D_2.

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Möbius semigroup Dynamics and the Fibonacci sequence

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We study the dynamics of semigroups of Mobius transformations on the Riemann sphere, focusing on the topology of the invariant structures that these systems generate, namely, their Fatou and Julia sets and attractors. We highlight the natural connections between the dynamics of rational functions, rational semigroups, and Mobius groups, as well as illustrate their differences with examples. In particular, we consider the motivating example, a one-parameter family of Mobius semigroups originating from a random dynamics variant of the Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21, ...

Joint work with David Fried (Boston University) and Sebastian M. Marotta (University of the Pacific).

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Invariant Potential Theory, Derivatives of Inner Functions, and $B^{p,q}$ Spaces in the Unit Ball of $\mathbb{C}^n$

Manfred Stoll

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Let $\mathbb{B}_n$ be the unit ball in $\mathbb{C}^n$ with boundary S. A bounded holomorphic function f on $\mathbb{B}_n$ is called an inner function if

$$\lim_{r\to 1^-}|f(rt)| = 1 \quad \text{for a.e. } t\in S.$$

For $0 < p < 1$ and $0 < q < \infty$, the space $B^{p,q}$ is defined as the space of holomorphic functions f on $\mathbb{B}_n$ for which

$$\|f\|^q_{B^{p,q}} = \int_{\mathbb{B}_n}(1-|z|^2)^{\frac{n}{p}-n-1}|f(z)|^q\,d\nu(z) < \infty,$$

where ν is normalized Lebesgue measure on $\mathbb{B}_n$. The case $q = 1$ gives the usual B^p spaces of holomorphic functions on $\mathbb{B}_n$. Using techniques and results of potential theory with respect to the Laplace Beltrami operator on $\mathbb{B}_n$, we prove that for $\frac{n}{n+q} < p < \min\{\frac{n}{q}, 1\}$, the radial derivative Rf of an inner function f satisfies

$$Rf\in B^{p,q} \Longleftrightarrow \int_{\mathbb{B}_n}(1-|z|^2)^{\frac{n}{p}-q-n-1}|\widetilde{\nabla} f(z)|^r d\nu(z) < \infty \tag{1}$$

for all r, q, with $1\le q\le r\le 2$, where $\widetilde{\nabla}$ denotes the gradient with respect to the Bergman metric on $\mathbb{B}_n$. As an application of (0.1) we prove the following.

Theorem: *Let f be an inner function on $\mathbb{B}_n$, $n\ge 1$.*
(a) *If $0<(1-\alpha)<\frac{1}{2}r$, $1\le r\le 2$, then*

$$Rf\in B^{\frac{n}{n+\alpha}} \Longleftrightarrow \int_{\mathbb{B}_n}(1-|z|^2)^{r+\alpha-2}|Rf(z)|^r d\nu(z) < \infty.$$

(b) *If $1\le q\le 2$, then for $\frac{n}{n+q} < p < \frac{n}{n+(q-1)}$ we have*

$$Rf\in B^{p,q} \quad \Longleftrightarrow \quad Rf\in B^s \text{ where } s = \frac{np}{n-p(q-1)}.$$

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On spaces of rational functions

Masahiko Taniguchi

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Usually, parameters such as critical points, critical values, and fixed points, are considered as standard parameters for the various deformation spaces induced from rational functions, such as the Hurwitz spaces, the Teichmüller spaces, and also the moduli spaces.
Here, I use the Bell spaces, or more generally, the Cauchy spaces of rational functions for the parameter spaces. We clarify the relations between the Bell spaces, or the Cauchy spaces, and the standard parameters as above. Also, we explain several natural kinds of the compactification of the deformation spaces as above.

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On value distribution of derivatives of meromorphic functions

Katsutoshi Yamanoi

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We discuss about value distribution of derivatives of meromorphic functions in the plane.

References

[1] K. Yamanoi, *Zeros of higher derivatives of meromorphic functions in the plane*, preprint.

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Monodromy property of analytic non integrable Hamiltonian system

Masafumi Yoshino

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In this talk we consider a resonant Hamiltonian system (E) $\dot{q} = \nabla_p H$, $\dot{p} = -\nabla_q H$, where $H = H(q,p)$ is a Hamiltonian function and $q = (q_1, \ldots, q_n)$ and $p = (p_1, \ldots, p_n)$ are the variables in $\mathbb{R}^n$ or in $\mathbb{C}^n$ $(n \geq 2)$. We denote the Hamiltonian vector field χ_H by $\chi_H := \{H, \cdot\}$, where $\{\cdot, \cdot\}$ denotes the Poisson bracket. We say that ϕ is the first integral of χ_H if $\chi_H \phi = 0$. The system (E) is said to be C^ω-Liouville integrable if there exist first integrals $\exists \phi_j \in C^\omega$ $(j = 1, \ldots, n)$ which are functionally independent on an open dense set and Poisson commuting, i.e., $\{\phi_j, \phi_k\} = 0$, $\{H, \phi_k\} = 0$. If $\phi_j \in C^\infty$ $(j = 1, \ldots, n)$, then we say C^∞- Liouville integrable.
Bolsinov and Taimanov (Invent. Math. 2000) showed the existence of a Hamiltonian system related with geodesic flow on a Riemannian manifold which is C^ω-nonintegrable as well as C^∞-integrable. They also showed that non C^ω- integrability is closely related

with the non Abelian property of a monodromy group. (See also Gorni, G. and Zampieri, G for a further extension.)
In order to study the monodromy structure of these C^{ω}-nonintegrable operators we note that, by the implicit function theorem, the integrability is essentially equivalent to the existence of an n-parameter solutions satisfying a certain nondegeneracy condition (complete solution). In view of the fundamental solution of a linear ordinary differential equation with irregular singularity these solutions are constructed via a formal exponential-log series, a power series with ordinary power series, exponentials and logarithm. We will construct first integrals in this form and we discuss asymptotic expansion, analytic continuation and monodromy.
Technically, we will make use of the moment sum of the series. Then we study the behaviors of the summed first integrals in a sector and analytic continuation beyond the sector.

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[2] Bolsinov, A.V. and Taimanov, I.A.: Invent Math. **140** (3), 639-650 (2000).

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Invited Talks

Deficient Divisors of Holomorphic Curves

Yoshihiro Aihara

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Let M be a smooth complex projective algebraic variety and $L \to M$ an ample line bundle. We let $\Gamma(M, L)$ denote the space of all holomorphic sections of $L \to M$. Let $W \subseteq \Gamma(M, L)$ be a linear subspace with $l_0 + 1 = \dim W \geq 2$. Denote by Λ the linear system determined by W, that is, $\Lambda = \mathbb{P}(W)$. The linear system Λ may have the non-empty base locus. Let $\mathcal{I}_0$ be the coherent ideal sheaf of the structure sheaf $\mathcal{O}_M$ over M that defines the base locus B_Λ of Λ as a complex analytic subspace.
Let $f : \mathbb{C} \to M$ be a transcendental holomorphic curve that is non-degenerate with respect to Λ, namely, the image of f is not contained in the support of any divisor in Λ. We let $\tilde{\delta}_f(D)$ and $\tilde{\delta}_f(B_\Lambda)$ denote modified deficiencies in the sense of Nochka. We consider the set $\widetilde{\mathfrak{D}}_f$ of deficient divisors defined by

$$\widetilde{\mathfrak{D}}_f = \{D \in \Lambda\,;\ \tilde{\delta}_f(D) > \tilde{\delta}_f(B_\Lambda)\}.$$

We can show that $\widetilde{\mathfrak{D}}_f$ is $\mathscr{P}$-polar in Λ. In particular, the Hausdorff dimensions of those sets are at most $2l_0 - 2$. We have a structure theorem for $\widetilde{\mathfrak{D}}_f$ as follows.

Theorem 1. *The set* $\widetilde{\mathfrak{D}}_f$ *is a union of at most countably many linear systems included in* Λ.

By the above theorem, we have a family $\{\Lambda_j\}$ of at most countably many linear systems in Λ such that $\widetilde{\mathfrak{D}}_f = \bigcup_j \Lambda_j$. We define $\mathfrak{L} = \{\Lambda_j\} \cup \{\Lambda\}$. Let $\tilde{\delta}_f(\Lambda)$ be the set of values of the function $\tilde{\delta}_f : \Lambda \to [0, 1]$. Then we can establish the correspondence between the values in $\tilde{\delta}_f(\Lambda)$ and the subfamilies of $\mathfrak{L}$.

Theorem 2. *The set* $\tilde{\delta}_f(\Lambda)$ *is an at most countable subset of* $[0, 1]$. *For each* $\alpha \in \tilde{\delta}_f(\Lambda)$, *there exists a unique finite subfamily* $\mathfrak{L}_\alpha = \{\Lambda_j^{(\alpha)}\}$ *of* $\mathfrak{L}$ *such that* $\alpha = \tilde{\delta}_f(B_{\Lambda_j^{(\alpha)}})$ *for all* $\Lambda_j^{(\alpha)} \in \mathfrak{L}_\alpha$ *and* $\alpha \neq \tilde{\delta}_f(B_{\Lambda_j})$ *for all* $\Lambda_j \in \mathfrak{L} \setminus \mathfrak{L}_\alpha$.

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Asymptotic Formulas and Inequalities for the multiple Gamma Functions

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There is an abundant literature on inequalities for the Gamma function Γ and its various related functions as well as their approximations. Only very recently, several authors began to investigate various inequalities for the double Gamma function Γ_2 and its approximation. Here, in this sequel to some of these recent works, we aim at presenting an integral representation of the triple Gamma function Γ_3, which is then used to derive an asymptotic formula for Γ_3. As a by-product of the results presented here, integral representations and asymptotic formulas for the Gamma function Γ and the double Gamma function Γ_2 are also given. The methods and techniques used in this paper can easily be extended to derive the corresponding integral representations and asymptotic formulas for the multiple Gamma functions Γ_n ($n \geqq 4$).
Also certain inequalities for the multiple Gamma functions Γ_n ($n = 2, 3, 4, 5$) and a more convenient explicit form of the multiple Gamma functions Γ_n ($n \in \mathbb{N}$), $\mathbb{N}$ being the set of positive integers, are presented.

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Conformal symmetries of the super Dirac operator

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We introduce the super Dirac operator acting on functions on $\mathbb{R}^{m|2n}$ with values in the super spinor space (see [1]). The definition is inspired by a construction of Stein and Weiss of $\mathfrak{o}(m)$-invariant generalized Cauchy-Riemann systems, see [4]. In this way we obtain an $\mathfrak{osp}(m|2n)$-invariant differential operator. This Dirac operator is a differential operator with values in a Clifford-Weyl algebra $\mathcal{C}l_{m|2n}$, which can be identified with endomorphisms on spinor space. The $\mathfrak{osp}(m|2n)$-action on the super spinors can then be embedded in this action of $\mathcal{C}l_{m|2n}$, see also [3]. All of this leads to a related $\mathfrak{osp}(m|2n)$-invariant Dirac operator on functions with values in the Clifford-Weyl algebra. For most statements the two Dirac operators can be identified.
Together with the super vector variable the Dirac operator generates the Lie superalgebra $\mathfrak{osp}(1|2)$, as in the classical $\mathbb{R}^m$-case. The square of the Dirac operator is for instance the well-known super Laplace operator. We prove the monogenic Fischer decomposition of spinor-valued super polynomials. This shows we obtain the Howe dual pair $(\mathfrak{osp}(m|2n), \mathfrak{osp}(1|2))$.
As in the classical case, see [2], the construction of Stein and Weiss leads to an operator which has a class of conformal symmetries that is larger than $\mathfrak{osp}(m|2n)$. We show that in the case of the super Dirac operator the conformal algebra is given by $\mathfrak{osp}(m+1, 1|2n)$.

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Separating transformation in extremal problems on non-overlapping domains

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Let $\mathbb{N}$, $\mathbb{R}$ be sets of natural and real numbers respectively, $\mathbb{C}$ be a complex plane, $\overline{\mathbb{C}} = \mathbb{C}\bigcup\{\infty\}$ be a one point compactification and $\mathbb{R}^+ = (0, \infty)$. Let $\chi(t) = \frac{1}{2}(t + t^{-1})$. Let $n \in \mathbb{N}$. A set of points $A_n := \left\{a_k \in \mathbb{C} : \ k = \overline{1, n}\right\}$, is called n **- radial** system, if $|a_k| \in \mathbb{R}^+$, $k = \overline{1, n}$, and $0 = \arg a_1 < \arg a_2 < \ldots < \arg a_n < 2\pi$. Denote $\alpha_k := \frac{1}{\pi} \arg \frac{a_{k+1}}{a_k}$, $\alpha_{n+1} := \alpha_1$, $k = \overline{1, n}$.
Let $r(B, a)$ be a inner radius of domain $B \subset \overline{\mathbb{C}}$, with respect to a point $a \in B$ (see [1, 2]). For an arbitrary n-radial system of points $A_n = \{a_k\}$ we assume that

$$\mathcal{L}(A_n) := \prod_{k=1}^{n} \left[\chi\left(\left|\frac{a_k}{a_{k+1}}\right|^{\frac{1}{2\alpha_k}}\right)\right] \cdot |a_k|.$$

Theorem 1. Let $0 < \gamma \leq \gamma_2$, $\gamma_2 = \frac{3}{5}$. Then for any 2-radial system of points $A_2 = \{a_k\}_{k=1}^{2}$ such that $\mathcal{L}\left(A_2\right) = 1$ and any system of non-overlapping domains B_0, B_1, B_2, B_∞ ($a_0 = 0 \in B_0 \subset \overline{\mathbb{C}}$, $\infty \in B_\infty \subset \overline{\mathbb{C}}$, $a_1 \in B_1 \subset \overline{\mathbb{C}}$, $a_2 \in B_2 \subset \overline{\mathbb{C}}$), we have inequality

$$\left[r\left(B_0, 0\right) r\left(B_\infty, \infty\right)\right]^\gamma r\left(B_1, a_1\right) r\left(B_2, a_2\right) \leqslant$$

$$\leqslant \left[r\left(\Lambda_0, 0\right) r\left(\Lambda_\infty, \infty\right)\right]^\gamma r\left(\Lambda_1, \lambda_1\right) r\left(\Lambda_2, \lambda_2\right),$$

where Λ_0, Λ_∞, Λ_1, Λ_2 and 0, ∞, λ_1, λ_2 are circular domains and poles of quadratic differential $Q(w)dw^2 = -\frac{\gamma w^4 + (4-2\gamma)w^2 + \gamma}{w^2(w^2-1)^2}\, dw^2$.

Joint work with A.K. Bakhtin (Institute of Mathematics of NAS of Ukraine).

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Periodicity of asymptotic Teichmüller modular transformations

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The quasiconformal mapping class group $\mathrm{MCG}(R)$ of a Riemann surface R induces the biholomorphic automorphism group of the Teichmüller space $T(R)$ and that of the asymptotic Teichmüller space $AT(R)$, which are defined as the Teichmüller modular group $\mathrm{Mod}(R)$ and the asymptotic Teichmüller modular group $\mathrm{Mod}_{AT}(R)$, respectively. A non-trivial element of $\mathrm{Mod}(R)$ is said to be *elliptic* if it has a fixed point in $T(R)$ and a non-trivial element of $\mathrm{Mod}_{AT}(R)$ is said to be *elliptic* if it has a fixed point in $AT(R)$. Every elliptic element of $\mathrm{Mod}(R)$ is realized as a conformal automorphism of the Riemann surface corresponding to its fixed point, and every elliptic element of $\mathrm{Mod}_{AT}(R)$ is realized as an asymptotically conformal automorphism of the Riemann surface corresponding to its fixed point.

If R is analytically finite, then an element of $\mathrm{Mod}(R)$ is elliptic if and only if it is periodic (finite order). This follows from the classical result of Nielsen. In the case where R is analytically infinite, an elliptic element of $\mathrm{Mod}(R)$ can be of infinite order. However, an elliptic element of $\mathrm{Mod}(R)$ induced by a conformal automorphism fixing a simple closed geodesic is periodic. On the other hand, even if R is analytically infinite, every periodic element of $\mathrm{Mod}(R)$ is elliptic.

In this talk, we consider the corresponding results for $\mathrm{Mod}_{AT}(R)$. We prove that, under the boundedness assumption on R, if an elliptic element of $\mathrm{Mod}_{AT}(R)$ is induced by an asymptotically conformal automorphism that fixes the free homotopy classes of infinitely many simple closed geodesics satisfying certain properties, then it is periodic. On the other hand, we also prove that every periodic element of $\mathrm{Mod}_{AT}(R)$ is elliptic under the same boundedness assumption on R.

Joint work with Katsuhiko Matsuzaki (Waseda University).

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The generalized Bell locus of rational functions and problems of Goldberg

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Let R be a rational map of degree d. Two rational maps R_1 and R_2 are said to be Möbius equivalent if there is a Möbius transformation $M : \hat{\mathbb{C}} \to \hat{\mathbb{C}}$ such that $R_2 = M \circ R_1$. Let X_d be the set of all equivalence classes of rational maps of degree d. Then, R has $2d-2$ critical points counted including multiplicity, and the set of critical points is invariant under taking a Möbius conjugate.
In [2], Goldberg showed the following theorem: *A $(2d-2)$-tuple B is the critical set of at most $C(d)$ classes in X_d, where $C(d)$ means the d-th Catalan number. The maximal is attained by a Zariski open subset of the space $\hat{\mathbb{C}}^{2d-2}$ of all B.* She also gave the problem that asks the explicit number of the equivalence classes corresponding to given critical set $c \in \hat{\mathbb{C}}^{2d-2}$.
In this talk, I give a complete answer to this problem when $d = 3$ and 4.

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Admissible boundary limits of Green potentials satisfying nonlinear inequalities in the unit ball of $\mathbb{C}^n$

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The classical theorem of Littlewood states that every Green potential in the unit disk has radial limit 0 almost everywhere on the unit circle. This was extended by Ullrich [3] to (complex) higher dimensions. It is not difficult to see that Green potentials do not necessarily have nontangential limits. Therefore Arsove and Huber [1] gave sufficient conditions for the density f that the Green potential of f has nontangential limits almost everywhere on the unit circle. The higher dimensional analogue was obtained by Cima and Stanton [2]. But the shape of approach regions in the higher dimensional case is different from the one dimensional case and includes tangential directions. Also, we should note that Cima and Stanton's result is not applicable to solutions of semilinear equations.
In this talk, we will discuss the existence of admissible boundary limits of Green potentials $u = Gf$ in the unit ball B of $\mathbb{C}^n$ which satisfy the nonlinear inequality

$$0 \leq f(z) \leq c(1 - |z|)^{n(p-1)} u(z)^p \quad \text{for a.e. } z \in B,$$

where $c > 0$ and $1 \leq p < n/(n-1)$ are constants.

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Quaternionic Fourier-Mellin Transform

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In this contribution we want to generalize the Fourier Mellin transform [1]

$$\forall(k,v) \in \mathbb{Z} \times \mathbb{R}, \quad \mathcal{M}_f(k,v) = \frac{1}{2\pi} \int_0^{\infty} \int_0^{2\pi} f(r,\theta) r^{-iv} e^{-ik\theta} d\theta \frac{dr}{r}, \tag{1}$$

where f denotes a function representing, e.g., a gray level image defined over a compact set of $\mathbb{R}^2$.
The quaternionic Fourier Mellin transform (QFMT) will be of the form

$$\forall(k,v) \in \mathbb{Z} \times \mathbb{R}, \quad \mathcal{M}_f(k,v) = \frac{1}{2\pi} \int_0^{\infty} \int_0^{2\pi} r^{-\mathbf{i}v} f(r,\theta) e^{-\mathbf{j}k\theta} d\theta \frac{dr}{r}, \tag{2}$$

where $f : \mathbb{R}^2 \to \mathbb{H}$ denotes a function from $\mathbb{R}^2$ into the algebra of quaternions $\mathbb{H}$, such that f is summable over $\mathbb{R}_+^* \times \mathbb{S}^1$ under the measure $d\theta \frac{dr}{r}$. $\mathbb{R}_+^*$ is the multiplicative group of positive and non-zero real numbers.
We will investigate the properties of the QFMT similar to the investigation of the quaternionic Fourier Transform (QFT) in [2, 3].

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Distortion theorems on some unit ball

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The object of this talk is to generalize the distortion theorems for convex mappings on finite dimensional Euclidean balls to infinite dimension, as well as improving some infinite dimensional results ([1]). From the perspective of the Riemann Mapping Theorem, an appropriate generalization of the open unit disc in the complex plane $\mathbb{C}$ would be the open unit ball B of a complex Banach space such that B is homogeneous. Indeed, it has been shown in [5] that every bounded symmetric domain in a complex Banach space is biholomorphically equivalent to such a ball. A key is some estimates involving the Möbius transformation and the Bergmann operator on a homogeneous ball which may be of some independent interest.
We refer to [2, 3] for reference and motivation for distortion results in higher dimensions.

Joint work with Cho Ho Chu(Queen Mary, University of London, England), Hidetaka Hamada (Kyushu Sangyo University, Japan) and Gabriela Kohr(Babeş-Bolyai University, Romania)

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Positive p-harmonic functions with zero boundary condition in the plane

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Let $1 < p < \infty$ and $0 < \phi < \pi$. We denote by D_ϕ the domain $\{z \in \mathbb{C} : |\arg z| < \phi\}$. We find positive p-harmonic functions $u(z)$ on D_ϕ with the boundary condition,

$$u(z) = \begin{cases} 0 & \text{for } |\arg z| = \phi \text{ or } z = 0, \\ \infty & \text{for } z = \infty, \end{cases} \tag{1}$$

or

$$u(z) = \begin{cases} 0 & \text{for } |\arg z| = \phi \text{ or } z = \infty, \\ \infty & \text{for } z = 0. \end{cases} \tag{2}$$

We consider the form $u(z) = r^k f(\theta)$ for $z = re^{i\theta}$. Aronsson [1] determined p-harmonic functions of the form $u(z) = r^k f(\theta)$, $2 < p < \infty$. If $u(z)$ is satisfied with the boundary condition (1), then k is positive. This k is denoted by k_+^p. On the other hand, if $u(z)$ is satisfied with the boundary condition (2), then k is negative. This k is denoted by k_-^p. Let $\beta = \pi/(2\phi)$. For $p = 2$, it is easy to calculate k_+^2 and k_-^2,

$$k_+^2 = \beta,\ k_-^2 = -\beta.$$

For $p \neq 2$, we consider two case, (i) $p > 2$ and (ii) $1 < p < 2$, and we give explicit representations for $u(z)$. k_+^p and k_-^p depend only on p and β. Moreover, we prove that

$$\lim_{p\to 2} k_+^p = k_+^2 = \beta,$$

and

$$\lim_{p\to 2} k_-^p = k_-^2 = -\beta.$$

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A Fréchet law for Maximal Cuspidal Windings

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We establish a Fréchet law for maximal cuspidal windings of the geodesic flow on a Riemannian surface associated with an arbitrary finitely generated, essentially free Fuchsian group with parabolic elements. This result extends previous work by Galambos [Gal72] and is obtained by applying Extreme Value Theory. Subsequently, we show that this law gives rise to an Erdős-Philipp law and to various generalised Khintchine-type results for maximal cuspidal windings. These results strengthen previous results by Sullivan, Stratmann and Velani for Kleinian groups, and extend earlier work by Philipp on continued fractions, which was inspired by a conjecture of Erdős.

Joint work with Marc Kesseböhmer and Bernd Stratmann (University of Bremen, Germany).

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On Chirka's question on the generalization of Forelli's theorem

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A *local* C^ℓ *singular foliation at* $p \in \mathbb{C}^n$ *by holomorphic discs* is a C^ℓ map $h : \Delta \times S^{2n-1} \to \mathbb{C}^n$ satisfying the followings:

(1) For each $v \in S^{2n-1}$, the correspondence $h(\cdot, v) : z \in \Delta \to h(z, v) \in \mathbb{C}^n$ is a holomorphic embedding.

(2) $h(0, v) = p$ for every $v \in S^{2n-1}$.

(3) The image $h(\Delta \times S^{2n-1})$ contains an open neighborhood of p in $\mathbb{C}^n$

(4) For each $v \in S^{2n-1}$, $\frac{\partial h}{\partial z}\big|_{(0,v)} = r_v v$ for some $r_v \in \mathbb{R}$.

(5) $h(z, e^{i\theta} v) = h(e^{i\theta} z, v)$ for every $\theta \in \mathbb{R}, z \in \Delta$ and $v \in S^{2n-1}$.

We prove the following theorem:

Theorem (J.-C. Joo, K. -T. Kim and G. Schmalz) *Let f be a function of class $C^1 \cap C^\infty(0)$ in a domain Ω containing the origin and let h be a local C^1 singular foliation at 0. Suppose that $f_v := f \circ h(\cdot, v)$ is holomorphic for every $v \in S^{2n-1}$. Then f is holomorphic in a neighborhood of the origin.*

This is a generalization of theorems of [2] and [1].

Joint work with K. -T. Kim (POSTECH) and G. Schmalz (University of New England).

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Zéros de la fonction holomorphe et bornée dans un polyhèdre analytique de $\mathbb{C}^2$.

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On cherche la condition nécessaire pour la surface S de zéros de la fonction holomorphe et bornée dans un polyhèdre analytique de l'espace $\mathbb{C}^2$ de (z_1, z_2).
Et pour la surface S vérifiant la condition, on construit la fonction $f(z_1, z_2)$ holomorphe et bornée, telle que S est définie par $f(z_1, z_2) = 0$.

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Numerical comparisons between Carathéodory measure hyperbolicity and positivity of canonical bundle

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In this talk, we will introduce several numerical relations between the Carathéodory measure hyperbolicity and the algebro-geometric positivity of the canonical bundle over a compact complex manifold. If possible, we will also mention the case of the cotangent bundle.
The Carathéodory measure hyperbolicity is defined by a positivity condition of the Carathéodory pseudo-volume form or the Carathéodory measure. These objects are intrinsic and modeled on the Poincaré measure on the complex ball and so they also have the volume decreasing property for holomorphic maps. It is a generalization of the classical Schwarz lemma. By the definition, it can be expected that a Carathéodory measure hyperbolic manifold is closely related to a manifold with a metric whose Ricci curvature is negative.

Hence it is very likely that the Carathéodory measure hyperbolicity leads to the positivity of the canonical bundle.
In order to connect these two notion to each other, we observe the curvature of the Carathéodory pseudo-volume form or the Carathéodory metirc which certainly reflects the Carathéodory measure hyperbolicity. We grasp the meaning of the curvatures of these objects in the sense of pluri-potential theory or real analysis, and investigate them. As a consequence, we can compare explicitly the quantities measuring the Carathéodory measure hyperbolicity with the quantities measuring the positivity of the canonical bundle. For example, we have that the curvature of the Carathéodory pseudo-volume form is bounded from above by -1. As its application, we can explicitly estimate the volume of the canonical bundle from below by the Carathéodory total volume.

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A generalized version of the Newlander-Nirenberg theorem

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For an almost complex manifold (M^{2m}, J), $m \geq 1$, there can be at most m independent J-holomorphic functions, which is the case that the integrability condition due to Newlander and Nirenberg as a complex version of the Frobenius theorem. In this talk, we determine the partial integrability on almost complex manifolds as a generalization of the Newlander-Nirenberg theorem. As a complex version of Cartan-Gardner theory, we follow the method which involves analyzing a certain torsion tensor.

This talk is based on the collaboration with Chong-Kyu Han (Seoul National University).

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Correspondence between spirallike functions and starlike functions

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Let λ be a real number with $-\pi/2 < \lambda < \pi/2$. In order to study λ-spirallike functions, it is natural to measure the angle according to λ-spirals. Thus we are led to the notion of λ-argument. This fits well the classical correspondence between λ-spirallike functions and starlike functions. Using this idea, we extend deep results of Pommerenke [2], [3] and Sheil-Small [4] on starlike functions to spirallike functions. As an application, we solved a problem given by Hansen in [1].

Joint work with Toshiyuki Sugawa (Tohoku University).

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Linear slices of the quasifuchsian space of punctured tori

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After fixing a marking (V, W) of a quasifuchsian punctured torus group G, the complex length λ_V and the complex twist $\tau_{V,W}$ parameters define a holomorphic embedding of the quasifuchsian space $\mathcal{QF}$ of punctured tori into $\mathbb{C}^2$. It is called the complex Fenchel-Nielsen coordinates of $\mathcal{QF}$. For $c \in \mathbb{C}$, let $\mathcal{Q}_{\gamma,c}$ be the affine subspace of $\mathbb{C}^2$ defined by the linear equation $\lambda_V = c$. Then we can consider the *linear slice* $\mathcal{L}_c$ of $\mathcal{QF}$ defined by $\mathcal{QF} \cap \mathcal{Q}_{\gamma,c}$ which is a holomorphic slice of $\mathcal{QF}$. For any positive real value c, $\mathcal{L}_c$ always contains the so called *Bers-Maskit slice* $\mathcal{B}M_{\gamma,c}$ studied in [1] and [2]. In this talk we consider the topology of $\mathcal{L}_c$ and show that if c is sufficiently small, then $\mathcal{L}_c$ coincides with $\mathcal{B}M_{\gamma,c}$ whereas $\mathcal{L}_c$ has other components besides $\mathcal{B}M_{\gamma,c}$ when c is sufficiently large [3].

Joint work with Yasushi Yamashita (Nara Women's University).

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Quaternion windowed linear canonical transforms

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In the talk, we introduce the windowed linear canonical transforms (WLCTs) of quaternion signals. It is named quaternion windowed linear canonical transforms (QWLCTs) of signals. The work is based on the recent paper in WLCTs of complex-valued signals. The generalized the results to the quaternion analysis setting. The QWLCTs offer local contents, enjoys high resolution, and eliminates cross terms. Some of their useful properties are derived, such as covariance property, orthogonality property and inversion formulas. The QWLCTs are new in the literature and has some consequences that are now under investigation.

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The IVP for potential vector field depending on time with more generalized governing rules

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This paper we discuss the influence of a governing law on a potential vector field, which always happens in a natural or technical phenomenons. This research leads us to the following initial value problem (IVP) of the type $\partial u/\partial t = Lu$, $u(0,.) = \varphi$ where t is the time variable, x is spacelike variable belonging to three - dimensional Euclidean space and L is a linear differential operator of first order (in the matrix - type); φ is the initial potential vector field. In the view of the classical Cauchy - Kovalevskaya theorem, the IVP is solvable provided that L has analytic coefficients and the initial function is analytic. On the other hand, the H. Levy example (see [1]) shows that there are equations of the above form with infinitely differentiable coefficients not having any solutions. The paper in hand describes all linear differential operators of the first order L, so that the IVP is uniquely solvable for every given potential vector field φ.

Joint work with Le Cuong (Hanoi University of Science and Technology) and Nguyen Thanh Van (Vietnam National University).

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Asymptotic expansion of Bergman kernel for tube domain of infinite type

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The asymptotic expansion formula for Bergman kernels near boundaries of smoothly bounded strongly pseudoconvex domains have wide and useful application to complex analysis. Its extension to smoothly bounded weakly pseudoconvex domains in general has been open. Kamimoto obtained such expansion formula for finite type pseudoconvex domains with circular symmetries and finite type pseudoconvex tube domains.([2] and [3])

Inspired by recent work by Bharali on growth estimate of the Bergman kernel for infinite type pseudoconvex domains ([1]), an expansion formula for Bergman kernel on the diagonal near exponentially-flat infinite type boundary point for pseudoconvex tube domain in $\mathbb{C}^2$ is obtained.

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Sums of products of Toeplitz and Hankel operators

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In this talk, we will consider a class of operators which are finite sums of products of Toeplitz and Hankel operators on the Dirichlet space of the unit disk. We then discuss recent results on the characterizing problems of when such an operator in that class is zero, of finite rank or compact.
This talk will be based on the recent works in [1] and [2].

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On meromorphic solutions of certain type of non-linear differential equations

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We consider meromorphic solutions of non-linear differential equation of the form

$$f^n + Q_d(z,f) = p_1(z)e^{\alpha_1(z)} + p_2(z)e^{\alpha_2(z)},$$

where $Q_d(z,f)$ is a differential polynomial in f of degree $d \leq n-2$ with rational functions as its coefficients, p_1, p_2 are rational functions and α_1, α_2 are polynomials. More precisely and mainly we have shown the conditions concerning $\frac{\alpha_1'}{\alpha_2'}$ that will ensure the existence and forms of the possible meromorphic solutions of the above equation. These results have extended and improved some known results obtained most recently.

Joint work with Chung-Chun Yang(China University of Petroleum) and Jian-Jun Zhang(Nanjing University).

A condition for an infinitely generated Schottky group to be classical

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Consider a set

$$\mathcal{C} = \{C_j, C'_j \mid j \in \mathbb{N}\}$$

of countably infinite number of pairs of simple closed curves in $\mathbb{C}$ such that not only these curves but also the interiors of them are mutually disjoint. We further assume that the exterior of C_j is mapped onto the interior of C'_l by a Möbius transformation g_j for every j. Let G be the group generated by all g_j defined as above. Here, we call G an *infinitely generated Schottky group* with respect to the loop family $\mathcal{C}$, if the limit set $\Lambda(G)$ of G is totally disconnected. If all elements of $\mathcal{C}$ are circles, then we call G an infinitely generated *classical* Schottky group.

We show that, if $\mathcal{C}$ satisfies the modified Maskit condition and the tameness condition, G is an infinitely generated Schottky group with respect to $\mathcal{C}$, further, if the corresponding Schottky marked Riemann surface R is maximally symmetric, G is an infinitely generated classical Schottky group.

Our consideration has a closed connection with a so-called circle domain theorem of Koebe, which has been generalized by He and Schramm.

Joint work with Masahiko Taniguchi (Department of Mathematics, Faculty of Science, Nara Woman University).

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Value distribution of bounded analytic functions

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Characterization of the value distribution of meromorphic functions is based on the growth of the Nevanlinna characteristic function $T(r, f)$, proximity function $m(r, a, f)$ and the counting function $N(r, a, f)$. Ahlfors-Simizu form of $T(r, f)$ is based on the spherical derivative and $m(r, a, f)$ can be described in terms of rotations of the Riemann sphere.
In case of bounded analytic functions situation is completely different. Growth of the spherical derivative does't provide some delicate information on the value distribution of a function. In order to study value distribution and boundary behavior of bounded analytic functions we will use behavior of the hyperbolic derivative and hyperbolic or pseudo-hyperbolic metric.
S. Yamashita was one of the first one who considered systematically hyperbolic function classes. He introduced hyperbolic Hardy, BMOA, Dirichlet and Lipschitz classes. Some research on hyperbolic Hardy classes was done by H.O. Kim, E.G. Kwon and etc. More recently, W. Smith studied hyperbolic little Bloch classes. Hyperbolic Q_p and Besov classes were studied by R. Zhao, S. Makhmutov, X. Li and etc.
In our talk we will present results concerning value distribution of bounded analytic functions and applications to composition operators. One of these results is the following:
THEOREM. *If φ is a bounded analytic function on the unit disk D, $a \in D$, and $0 < r < 1$, then*

$$T_\star(r, \varphi) = m_\star(r, a, \varphi) - N(r, a, \varphi)$$

where

$$T_\star(r, \varphi) = \frac{1}{\pi} \iint_{|z|<r} \left(\frac{|\varphi'(z)|}{1 - |\varphi(z)|^2} \right)^2 \ln \frac{r}{|z|} dA(z)$$

and $m_\star(r, a, \varphi)$ is a hyperbolic proximity function.

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Schwarzian derivatives of asymptotically conformal extension of univalent functions

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Let $\mathbb{D}$ be the unit disk and $\mathbb{D}^* = \mathbb{C} - \overline{\mathbb{D}}$. For a Beltrami coefficient $\mu \in L^\infty(\mathbb{C})$ with $\|\mu\| < 1$ such that $\mu(z) = 0$ for $z \in \mathbb{D}$, let f^μ be a quasiconformal automorphism of $\mathbb{C}$ with $f^\mu_{\bar{z}}/f^\mu_z = \mu(z)$. The restriction of f^μ to $\mathbb{D}$ is conformal, that is, $f^\mu|_{\mathbb{D}}$ is a univalent function on $\mathbb{D}$. We consider its Schwarzian derivative $\varphi_\mu(\zeta) = S_{f^\mu}(\zeta)$. On the other hand, $f^\mu|_{\mathbb{D}^*}$ is a quasiconformal homeomorphism, and if $\lim_{|z|\to 1+0} \mu(z) = 0$, then $f^\mu|_{\mathbb{D}^*}$ or $\mu|_{\mathbb{D}^*}$ is called asymptotically conformal. In this case, the corresponding Schwarzian derivative satisfies $\lim_{|\zeta|\to 1-0}(1 - |\zeta|)^2|\varphi_\mu(\zeta)| = 0$. We investigate a quantitative estimate of this decay order in terms of that for μ. Set

$$k(t) = \sup_{1<|z|\le 1+t} |\mu(z)| \ ; \qquad \sigma(t) = \sup_{1>|\zeta|\ge 1-t} (1 - |\zeta|)^2 |\varphi_\mu(\zeta)|.$$

Becker [1] proved that, for every $\varepsilon > 0$, it holds that

$$\sigma(t^{1+\varepsilon}) \leq 6\{k(t) + t^{2\varepsilon}\} \qquad (0 < t < 1).$$

Restricting ourselves to the case where $k(t) = O(t^\alpha)$, we have the following:

Theorem. *For every α with $0 < \alpha < 1$, there exists a constant $C > 0$ depending only on α such that, if $k(t) \leq c\,t^\alpha$ then $\sigma(t) \leq Cc\,t^\alpha$.*

A similar result holds by replacing $\sigma(t)$ with

$$\beta(t) = \sup_{1>|\zeta|\geq 1-t} (1 - |\zeta|)|\phi_\mu(\zeta)|,$$

where $\phi(\zeta) = (f^\mu)''(\zeta)/(f^\mu)'(\zeta)$ is the pre-Schwarzian derivative of $f^\mu|_{\mathbb{D}}$.

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Morrey capacity of balls

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The notion of classical Newton capacity has been generalized to various forms. Among others, Meyers introduced a general notion of L^p-capacity, which is defined by general potentials of functions in the Lebesgue space L^p and such notion of capacity has been proved to provide rich results in the nonlinear potential theory as well as in the study of various function spaces and partial differential equations. The most useful L^p-capacity is Riesz capacity. The aim in this note is to estimate the Riesz capacity of balls $B(x, r)$ centered at x of radius r in the Orlicz setting.

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Boundedness of Berezin transform on Herz spaces

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In this paper, we give the condition for the boundedness of the Berezin transforms on Herz spaces with a normal weight on the unit ball of $\mathbb{C}^n$. And we provide the integral estimates concerning pluriharmonic kernel functions. Using this, we finally obtain the growth estimates of the Berezin transforms on such Herz spaces.

Joint work with Chu-Hee Cho (Seoul National University).

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Parametrizations of Teichmüller spaces by trace functions

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Let $\bar{S}$ be a surface of type (g, m), that is, a smooth compact and orientable surface of genus g with m boundary curves, and S the interior of $\bar{S}$, where $2g-2+m > 0$. The Teichmüller space $\mathcal{T}(g, m)$ is the space of equivalence classes of marked complete hyperbolic metrics

of curvature -1 on S such that each boundary curve is homotopic to a unique closed geodesic curve in S.
There are finitely many closed curves $c_1, \dots, c_N$ on S such that their geodesic length functions give a global real analytic coordinate system of $\mathcal{T}(g,m)$. It is known that the minimal number of geodesic length functions needed to parametrize globally $\mathcal{T}(g,m)$ is equal to $6g-6+3m$ for $m>0$; $6g-5$ for $m=0$, whereas $\mathcal{T}(g,m)$ is homeomorphic to $\mathbb{R}^{6g-6+3m}$. In this talk we establish a trace identity for groups of type $(1,2)$ and use this identity to find an algebraic equation satisfied by $6g-5$ geodesic length functions, or equivalently, trace functions, which parametrizes $\mathcal{T}(g,0)$. We basically follow Feng Luo's paper [1] which gave a clear account of the parametrization of the Teichmüller spaces the minimal number of geodesic length functions.

Joint work with Toshihiro Nakanishi (Shimane University).

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Infinite dimensional manifolds with properties similar to Stein manifolds

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OKa [4] proved that a pseudoconvex Riemann domain over $\boldsymbol{C}^n$ is holomorphically convex , holomorphically separated and so a domain of holomorphy by Cartan-Thullen theorem. A complex manifold, which is holomorphically convex and holomorphically separated, is called a Stein manifold. The problem to ask if a given complex manifold is a Stein manifold has been investigated in various situation(cf. Siu [5] and its References etc.). Moreover the result of Oka has been extended to Riemann domains over various infinite dimensional topological vector spaces(cf. Dineen [1] and their References etc.).
Let E be a complex Banach space with a Schauder basis and let $G(E;\, r)$ be the Grassmann manifold of all r-dimensional complex linear subspaces in E. Let (ω, φ) be a pseudoconvex Riemann domain over $G(E;\, r)$ with $\omega \neq G(E;\, r)$.
In this talk we prove that the Riemann domain (ω, φ) is holomorphically convex, holomorphically separated and a domain of holomorphy. This result gives an example of infinite dimensional complex manifolds with properties similar to Stein manifolds.

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Concave functions in the geometric function theory

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Similar to convex, close-to-convex or starlike functions, *concave functions* form a special class in the geometric function theory. In an early stage A.W. Goodman for instance considered them in a general way (see [1]). However the first detailed analysis was given by A.E. Livingston in 1994 in [2]. Based on the work of Livingston, F.G. Avkhadiev and K.-J. Wirths continued the analysis of concave functions in [3]-[6], focusing on the coefficients of Taylor and Laurent expansions.
This presentation will introduce some basic analytic characteristics of concave functions and give a short summary of known estimates for the coefficients. Furthermore several questions concerning these results, as well as some future tasks will be mentioned.

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The repelling density problem and logarithmic equidistribution in non-archimedean and complex dynamics

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It is an open problem whether the repelling periodic points are dense in the classical Julia set of a rational function over non-archimedean fields. In this talk, we give a partial positive answer to this question based on a study of "logarithmic equidistribution" on Berkovich projective line over non-archimedean and complex number fields.

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Reflection Group of type D_4 and Holonomic Systems with Singularities along its Discriminant Locus

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In my talk, after defining the discriminant of the reflection group of type D_4, I introduce systems of uniformization equations with singularities along its zero locus. My main purpose is to solve a special case of such a system by elliptic integral and hypergeometric functions.

Related topics are discussed in [1], [2], [3] and the references there.

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On a theorem of Paul Yang

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Paul Yang showed in [1] that, given any two negative constants, the bidisc cannot admit any complete metric with its holomorphic bisectional curvature bounded between. His proof works for any dimensional polydiscs as well as the Hermitian symmetric domains with rank not less than 2. This method works also for product of complete Kälerian manifolds, as shown by H. Seshadri and F. Zheng [2]. Still, whether there are examples falling into this category that are neither homogeneous nor product is poorly understood. In this talk, I will present new examples of inhomogeneous bounded domains that cannot admit complete Kähler metrics with their bisectional curvature bounded between any prescribed negative bounds.

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On the coefficients of the Riemann mapping function for the complement of the Mandelbrot set

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It is well known that the Mandelbort set $\mathbb{M} := \{c \in \mathbb{C} : c, c^2 + c, (c^2 + c)^2 + c, \cdots \not\to \infty \text{ as } n \to \infty\}$ is connected, but its local connectivity is still unknown. Douady and Hubbard demonstrated the connectedness of the Mandelbrot set by constructing a conformal isomorphism $\Phi : \widehat{\mathbb{C}} \setminus \mathbb{M} \to \widehat{\mathbb{C}} \setminus \overline{\mathbb{D}}$ where $\widehat{\mathbb{C}}$ is the Riemann sphere and $\overline{\mathbb{D}}$ is the closed unit disk. According to Carathéodory's continuity theorem, the map $\Psi := \Phi^{-1}$ extends continuously to the unit circle, which implies that the Mandelbrot set is locally connected. This is the motivation of our study. Jungreis has presented an algorithm to compute the coefficients b_m of the Laurent series of $\Psi(z)$ at ∞. Several detailed studies of the coefficients b_m were given in [1], and others. There are also several empirical observations by Zagier mentioned in [1]. After that, Komori and Yamashita studied a generalization of b_m in [3]. Furthermore Ewing and Schober studied the coefficients a_m of the Taylor series of the function $f(z) := 1/\Psi(1/z)$ at the origin in [2].
In this talk we denote several properties of a generalization of the coefficients a_m and b_m. Specifically a formula for these coefficients are given. Infinitely many coefficients are zero and also infinitely many non-zero coefficients are determined. During the presentation, we touch the observations by Zagier as well.

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Repesentation of integral fomulas on Clifford analysis

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We give a representation form of integral formulas and research properties of the kernel formula on Clifford analysis.

Joint work with Sujin Lim (Pusan National University).

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Power deformations of univalent functions

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For an analytic function $f(z)$ on the unit disk $|z| < 1$ in the complex plane with $f(0) = f'(0) - 1 = 0$ and $f(z) \neq 0, 0 < |z| < 1$, we consider the power deformation $f_c(z) = z(f(z)/z)^c$ for a complex number c. We determine those values c for which the operator $f \mapsto f_c$ maps a specified class of univalent functions into the class of univalent functions. A little surprisingly, we will see that the set is described by the variability region of the quantity $zf'(z)/f(z),\ |z| < 1$, for most of the classes that we consider in the present talk. As an unexpected by-product, we show boundedness of strongly spirallike functions.

Joint work with Yong Chan Kim (Yeungnam University).

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Stability and bifurcation in random complex dynamics

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Regarding the dynamics of a rational map h with $\deg(h) \geq 2$ on the Riemann sphere $\hat{\mathbb{C}}$, we have the chaotic part in $\hat{\mathbb{C}}$. In fact, in the Julia set of h, which is non-empty, we have the chaos in the sense of Devaney.
However, we show ([1, 2]) that in the (i.i.d.) random dynamics of polynomials on $\hat{\mathbb{C}}$, generically, we have all of the following (1)(2)(3).

(1) The chaos of the averaged system disappears, due to the automatic cooperation of many kinds of maps in the system (**cooperation principle**), even though each map of the system or each pathwise dynamics has a chaotic part.

(2) The limit state is stable under perturbations of the system.

(3) The speed of convergence to the limit state is exponentially fast.

We also investigate the bifurcation of the limit states under the assumption that the "kernel Julia set" of the systems are empty.
Moreover, we see that even though the chaos of the averaged system disappears in the sense of the space of C^0 functions, we might have a kind of chaos in the sense of differentials or in the sense of the space of Hölder continuous functions. More precisely, if we consider the function T_∞ of probability of tending to $\infty \in \hat{\mathbb{C}}$, then under certain conditions, T_∞ is continuous on $\hat{\mathbb{C}}$ but varies only on a very thin fractal set (so called the Julia set of the associated semigroup), and the pointwise Hölder exponent of T_∞ is strictly less than 1 for a.e. point in the Julia set with respect to some nice "invariant" measure. In such case, the function T_∞ is called the **devil's coliseum**.

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A representation and an interpolation theory for harmonic Bergman functions

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Let Ω be a smooth bounded domain in the n-dimensional Euclidean space $\mathbb{R}^n$. For $1 \leq p < \infty$, we denote by $b^p(\Omega)$ the harmonic Bergman space on Ω. It is a fact [4] that if $1 < p < \infty$, any $f \in b^p(\Omega)$ has the following representation:

$$f(x) = \sum_{i=1}^{\infty} a_i R(x, \lambda_i) r(\lambda_i)^{(1-\frac{1}{p})n}, \tag{3}$$

for some $\{\lambda_i\}_i \in \Omega$ and $\{a_i\} \in \ell^p$, where $R(x, y)$ denote the harmonic Bergman kernel and $r(x)$ denotes the distance between x and $\partial\Omega$.
In the present talk, we give another representation by using modified reproducing kernels in order to handle the case $p = 1$. We remark that modified kernels are introduced in [1]. We also discuss interpolating sequences (cf. [2]).

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On the growth functions of hyperbolic Coxeter groups

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We will talk about the growth functions of the Coxeter groups, which are known to be rational functions (c.f. [1, 2]). In particular we will study the distributions of poles of the growth functions of simplex hyperbolic Coxeter groups (c.f. [3]).

This is a joint work with Yohei Komori (OCAMI and Osaka City University).

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Some problems related to pseudo differential equations in non-smooth domains

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One considers the solvability of the equation

$$(Au)(x) = f(x), \quad x \in M,$$

where A is a pseudo differential operator with symbol $A(x,\xi), M \subset \mathbf{R}^m$ is a bounded domain with a boundary having the singularities of "cone" or "wedge" type.
Earlier the author introduced the concept of wave factorization for the symbol $A(\xi)$ for elliptic operator A. It was connected with multi-dimensional complex domain (radial tube domain over cone) and requirement of analyticity for factors. (We refer to [1] and [2] for details). This approach permitted to describe full solvability cases for a model pseudo differential equation in the cone

$$C_+^a = \{x \in \mathbf{R}^m : |x'| > a|x_m|,\ a > 0\},\ x = (x_1, x_2, ..., x_m),\ x' = (x_1, ..., x_{m-1}).$$

In this report we consider the situations, when the cone C_+^a can be degenerated into a ray, i.e. $a \to +\infty$, and analyze possibilities for studying solvability for such model equations.

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